SIMULATION USING IN SITU ADAPTIVE TABULATION AND FAST FLUID DYNAMICS

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ABSTRACT

Ventilation with stratified air distribution is commonly used to improve building energy efficiency and indoor environment quality. A fast indoor airflow simulation can be useful for the ventilation design, performance evaluation, and model predictive ventilation control. As an intermediate model between computational fluid dynamics (CFD) and multizone models, a fast fluid dynamics (FFD) was proposed to balance simulation accuracy and computing speed. In this paper, we propose to further speed up the FFD simulation by using a computation reduction technique called in situ adaptive tabulation (ISAT). ISAT is a general function approximation method and was first proposed to speed up CFD simulation for combustion. Using ISAT, we are able to store the key FFD simulation data in a table and retrieve the data from the table for simulations with similar boundary conditions. This paper presents our ISAT-FFD implementation and some preliminary results. In a parametric study with 60,000 simulations, we showed that the ISAT-FFD simulation could compute the key indoor environment data for a natural convection flow at a speed up of 50 times faster than the FFD simulation and the prediction errors are within 1K. In the other case study, we showed that a trained ISAT-FFD model can predict the key environmental data by simply retrieving from the data table with controllable accuracy and little computing time. The ISAT-FFD can also properly handle the scenario when the independent variables are outside pre-trained data range.

INTRODUCTION

Ventilation with stratified air distribution may provide better building energy efficiency and indoor air quality (Yuan, et al. 1999). Examples are displacement ventilation, underfloor ventilation, personal ventilation and natural ventilation. A fast indoor airflow simulation can be useful for the ventilation design, performance evaluation, and model predictive ventilation control. As multizone airflow network models assume the air in the room to be well-mixed, they are not suitable for the simulation of stratified airflows. By solving the Navier-Stokes equations and other governing equations for energy and species, Computational Fluid Dynamics (CFD), can be used to support the design and performance evaluation of the ventilations with stratified air distribution. However, it is time consuming to run the CFD simulations, which becomes an obstacle if one wants to use the computer simulation for building design, emergency management (such as fire emergency evacuation), and model predictive control of building ventilation.

To address the speed limit of CFD, a Fast Fluid Dynamics (FFD) model (Zuo, et al. 2009, Zuo 2010, Zuo, et al. 2010d) was proposed as an intermediate method between the CFD and multizone airflow network method. FFD solves the same Navier-Stokes equations and governing equations for energy and species as CFD does. By using different mathematical algorithms, which sacrifice some accuracy, FFD gains a significant enhancement in computing speed compared to CFD (Zuo, et al. 2009, Zuo 2010, Zuo, et al. 2010d). In addition, the FFD simulation was further accelerated by running in parallel on graphics processing unit (Zuo, et al. 2010a). FFD has been used to simulate various airflows inside and around the buildings (Zuo, et al. 2009, Zuo 2010, Zuo, et al. 2010c, Zuo, et al. 2010b, Zuo, et al. 2010a, Zuo, et al. 2010d, Jin, et al. 2012, Jin, et al. 2013) and it was coupled with the Modelica buildings library (Wetter, et al. 2014) for integrated simulation of indoor environment and building HVAC systems (Zuo, et al. 2014, Wetter, et al. 2015, Zuo, et al. 2015). Although significantly faster than CFD, FFD is still not fast enough for some applications. For instance, model predictive control of building ventilation requires the model to predict critical indoor environment parameters in a fast manner such that the time for each optimization is low as hundreds of iterations are needed to find the optimal controls.

Another approach to speed up the CFD simulations is to develop regression models based on pre-computed CFD results. Although it may take some time to run some CFD simulations to generate data for model training purpose, those regression models that are usually in low order can compute the output almost instantaneously by...
interpolating or extrapolating from the existing data set. For instance, Zhou, et al. (2009) developed an Artificial Neural Network (ANN) model based on CFD simulation for the office ventilation system design and operation. Li, et al. (2013) developed a reduced order model for the simulation and control of indoor thermal environment using a proper orthogonal decomposition based on CFD results. Although the regression model is fast, they can perform well only when the model input is within the range of training data provided by the pre-computed CFD results. If the model input is outside the range, extrapolation may be used to get the output but without a guarantee on accuracy.

One method to address the limited range of model inputs is to call the CFD simulation during the regression process if the inputs is out of range pre-defined by the training data. Then we can use the newly generated data to train the regression model. This is the basic idea of in situ adaptive tabulation (ISAT). ISAT is a general function approximation method and was first proposed to speed up CFD simulations for combustion (Pope 1997). ISAT stores the key simulation data in a data table and retrieves the data from the table if the output of certain model inputs (queries) can be read or interpolated using the stored data to meet the specified error tolerance. If not, the ISAT algorithm will run a CFD simulation and store the new data into the table. The ISAT has mainly been used in accelerating combustion related simulation, such as solving reaction-diffusion equation (Singer, et al. 2004) and modeling reacting flow with detailed chemistry (Singer, et al. 2006). It was also extended to model predictive control for chemical processes (Hedengren, et al. 2008).

The above literature review shows that it is possible to develop a self-learning regression model for indoor airflows by combing the FFD and ISAT. Since the ISAT has never been used for the indoor airflow simulation, it is not clear if this combination would work and what the accuracy and speed can be. This paper is to answer those questions. In the rest of the paper, we will briefly introduce FFD and ISAT. Then we will discuss the implementation of ISAT-FFD. After that, we will show some results of our ISAT-FFD program using two cases studies. At the end, we will discuss the limitations of current work and future research opportunities, and present conclusions.

**METHODOLOGY**

**Fast Fluid Dynamics**

Similar to CFD, FFD also solves the Naiver-Stokes equations

\[
\frac{\partial U_i}{\partial t} = -U_j \frac{\partial U_i}{\partial x_j} + v \frac{\partial^2 U_i}{\partial x_j^2} - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + F_i, \tag{1}
\]

where \(i,j \in \{1,2,3\}\) are the indices, \(U_i\) and \(U_j\) are velocity components in \(x_i\) and \(x_j\) directions, respectively, \(v\) is kinematic viscosity, \(\rho\) is fluid density, \(P\) is pressure, \(t\) is time and \(F_i\) is the source term, such as buoyancy force. To solve the Navier-Stokes equations, FFD splits it into the following three equations:

\[
\frac{\partial U_i}{\partial t} = -U_j \frac{\partial U_i}{\partial x_j}, \tag{2}
\]

\[
\frac{\partial U_i}{\partial t} = v \frac{\partial^2 U_i}{\partial x_j^2} + F_i, \tag{3}
\]

\[
\frac{\partial U_i}{\partial t} = -\frac{1}{\rho} \frac{\partial P}{\partial x_i}, \tag{4}
\]

FFD first solves the advection equation (2) using a semi-Lagrangian method (Courant, et al. 1952). Then it solves the diffusion equation (3) with an implicit scheme. After that, it solves the pressure equation (4) together with the continuity equation

\[
\frac{\partial U_i}{\partial x_i} = 0, \tag{5}
\]


**In Situ Adaptive Tabulation**

For a nonlinear model

\[
y = f(x), \tag{6}
\]

where \(x\) is a vector of independent variables in \(R^m\), and \(y\) is vector of dependent variables in \(R^n\), ISAT provides linear estimations as:

\[
y^* = g(x, x_1), \tag{7}
\]

\[
\|B(y^* - y)\| \leq \varepsilon_{tol}, \tag{8}
\]

where \(x_1\) is a vector of independent variables in \(R^m\) and is stored in the data table, \(y^*\) is a linear estimation of \(y\), \(\varepsilon_{tol} \geq 0\) is specified error tolerance, and \(B\) is a scaling matrix. A region of accuracy (ROA) around \(x_1\) is defined if for any \(x\) within the ROA, the linear model (7) can calculate the \(y^*\) that meets condition (8).

When a new \(x_q\) is given, ISAT will perform one of the following three actions:
1. **Retrieve**: if \( x_q \) is within a ROA of a stored \( x_t \), then the linear model (7) is applied to retrieve the value of \( y_q^* \) (Figure 1a).

2. **Grow**: If \( x_q \) is outside the ROA, ISAT uses the nonlinear model (6) to calculate the exact solution \( y_q \) and the linear model (7) for the estimated solution \( y_q^* \). Depending on how \( y_q \) and \( y_q^* \) meet the condition (8), two actions will be taken. If the condition (8) is met, the ROA will grow to include \( x_q \) (Figure 1b). In this case, no new data record in the data table will be created. Instead, we just grow the ROA of an existing data record in the data table. Otherwise, an operation Add described below will be done.

3. **Add**: If \( x_q \) is outside the ROA and the condition (8) is not met, a new data record for a new ROA around \( x_q \) will be added (Figure 1c). In this case, we add a new record into the data table.

The key of the ISAT is how to construct the data table and design the mechanism to efficiently perform “retrieve”, “grow”, and “add” actions when the table is enormously large. A series of developments have been done which are available in the literature (Pope 1997, Singer, et al. 2004, Singer, et al. 2006). Based on those research, this study attempts to speed up indoor airflow simulation by integrating ISAT and FFD.

**ISAT-FFD Integration**

Figure 2 shows the workflow of our implementation of integrated ISAT-FFD. From the top, a query (input data) is sent to the ISAT-FFD. Then, the ISAT-FFD will check whether the input data is within a ROA of any stored input data records in the table. If within the ROA, the retrieval function will be called to retrieve the data. If not, the FFD simulation will be launched to compute the result. After getting the FFD simulation data, ISAT will check if the error is less than the specified error tolerance. If so, the existing record will grow to expand the ROA to a broader range. If not, a new record will be added to create a new ROA. In both cases, the exact results by FFD will be returned.

This study uses the ISAT source code provided by the Cornell University which is available at [https://tcg.mae.cornell.edu/isat.html](https://tcg.mae.cornell.edu/isat.html). We have revised some codes for compatibility since the ISAT libraries are written in the FORTRAN language and the FFD program is written in the C language. An interface between the ISAT and FFD was also implemented for the purpose of data exchange.

**CASE STUDIES**

To evaluate the implementation of the ISAT-FFD program, we simulated a natural convection flow and a mixed convection flow. In the first case, we focused on parametric simulation studies by starting with an empty data table. In the second case, we first trained the ISAT-FFD, then evaluated its performance for queries both inside and outside the training data set. Both simulations were performed using a computer with an Intel Xeon Processor E5-1603 with a four-core CPU at 2.8 GHz. Although the CPU has multiple cores, the ISAT-FFD simulation used only a single core.
Natural Convection in an Empty Room

As shown in Figure 3, there is a natural convection flow in a 1 m × 1 m × 1 m empty room. The temperatures \( T_W \) and \( T_E \) on the west and east walls are prescribed. For the other walls, the floor, and the ceiling, adiabatic boundary conditions are applied, hence the heat flow is zero. The ISAT-FFD model is used to predict the velocity and temperature at the center of the room (0.5 m, 0.5 m, 0.5 m) with different \( T_W \) and \( T_E \) ranging from 20°C to 30°C. This is a simplified case with only two model inputs for the purpose of model evaluation. In reality, we will have more than two inputs, such as temperatures of all walls, temperatures and air velocities of different inlets, and internal heat gains.

A uniform 20 × 20 × 20 grid is used. The FFD model and numerical settings for this flow have been validated in a previous study (Zuo, et al. 2014). For the ISAT, the query consists of two data elements \( T_W \) and \( T_E \) and the output includes four data elements: three velocity components and temperature at the center of the room. The output data are time-averaged after the flow is fully developed.

As expected, the time of retrieve actions is negligible compared to the time of grow and add actions which require a FFD simulation. Thus, we can see that the time growth mainly happens during the first 5,000 queries when add and grow actions are conducted to populate the data table or train the model. After the model is trained, most queries can be responded by retrieve actions so that the growth of total computing time significantly slows down and becomes almost flat for most settings after 20,000 queries. Thus, when we use a well-trained ISAT-FFD program in the model predictive control, the computing speed won’t be an issue anymore, just as those regression models do.

Mixed Ventilation with a Heated Box inside the Room

This case is a mixed ventilation for a space (2.44 m × 2.44 m × 2.44 m) with a heated rectangular box (1.22 m × 1.22 m × 1.22 m) inside (Figure 5). The center of box is located at (1.22 m, 1.22 m, 0.61 m). The inlet is located on the west wall with a height of 0.03 m and the outlet on the east wall with a height of 0.08 m. The velocity and temperature of the inlet flow are 0.455 m/s and 22.2°C, respectively. The default temperature is 25.8°C on the ceiling, 26.9°C on the floor and 27.4°C on other walls. The temperature on the surface of the box is 36.7°C. The flow structure is complex because the internal obstacle and the airflow is under the strong interaction of inertia force and buoyance force.
In this case, the ISAT-FFD model is used to predict the averaged occupied zone (\( Z \leq 1.22 \text{ m} \)) temperature \( T_{\text{occ}} \) with different wall temperatures on the west and east (\( T_W \) and \( T_E \)). The FFD simulation adopted a non-uniform \( 20 \times 20 \times 20 \) mesh. The FFD model and settings were validated against the experimental data (Wang, et al. 2009) in a previous study (Zuo, et al. 2015).

![Figure 5 Schematic of the mixed convection in an empty room with a heated box](image)

We first trained the ISAT model using 10,000 queries, which is a combination of \( T_W \) and \( T_E \) uniformly distributed from 20°C to 30°C with an increment of 0.1 K. The specified error tolerance for \( T_{\text{occ}} \) is 0.5 K. After the training period, we evaluated the performance of ISAT-FFD with 50 queries. The queries are made of a uniform distribution of \( T_W \) and \( T_E \) from 15°C to 35°C with an increment of 1 K. Thus, 25% of queries are within the training data range and 75% are outside.

Figures 6 and 7 show the prediction error as a function of query inputs and corresponding actions taken by ISAT (retrieve, grow or add). The prediction error is the difference in computed \( T_{\text{occ}} \) between the ISAT-FFD and FFD with the same inputs. Queries that are outside the training domain and far away from the training domain boundary lead to either a grow or an add action which requires a FFD simulation to produce an accurate solution with a zero error. Queries that lay in the training range or out of training range but close to the boundary have a non-zero error because ISAT retrieves the values from the stored table without an FFD simulation. Some retrieved points, which are outside the training range but close to the boundary, have errors between 0.5-0.6 K, although the error tolerance is 0.5 K. This shows that the error control algorithm could under-estimate the error. This is mainly caused by the linear approximation used in the error estimation while the real model is nonlinear.

**DISCUSSION**

The case studies shows that ISAT-FFD has great potential for model predictive control of indoor environment due to it high speed and capability to handle inputs outside training domain. However, in ISAT, the data table can change from one simulation to another as the table expands by the grow and add actions. This may introduce a discontinuity in the cost function in the optimization that can cause gradients to not exist and their numerical approximations may have a large error. Thus, it may cause numerical problems if ISAT-FFD is used for model predictive control with gradient-based optimization algorithm. To avoid this problem, one should resort to use an optimization algorithm that does not require differentiability of the cost function.

The computing speed of ISAT-FFD can be further increased. Since the time of retrieve action is negligible compared to the time of grow and add actions, the key of speeding up ISAT-FFD is to accelerate the grow and add actions. To further accelerate the ISAT_FFD simulation, we can accelerate the FFD simulation directly. Since
most of the computing time is used by the FFD simulation, we can also speed up the ISAT-FFD simulation is to run the FFD in parallel on a multiple core CPU or computer graphics processing units (Zuo, et al. 2010a).

CONCLUSION
This study proposed a fast indoor airflow simulation method by speeding the FFD simulation with the ISAT algorithm. Our results show that

- In preliminary case study with two inputs and four outputs, the ISAT-FFD could accelerates the FFD simulation up to 50 times which can be used to accelerate parametric studies of indoor environment simulations.
- After the ISAT-FFD model is trained, it can predict the indoor environment almost instantaneously since only data retrieved is needed.
- The ISAT-FFD model can satisfactorily handle model inputs when they are out of training domain. However, the error estimator based on linear approximations may underestimate the prediction error.
- The speed of ISAT-FFD simulation can be further accelerated through parallel computing.

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